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RESEARCH ARTICLE

An identification model of urban critical links with macroscopic fundamental diagram theory

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Abstract How to identify the critical links of the urban road network for actual traffic management and intelligent transportation control is an urgent problem, especially in the congestion environment. Most previous methods focus on traffic static characteristics for traffic planning and design. However, actual traffic management and intelligent control need to identify relevant sections by dynamic traffic information for solving the problems of variable transportation system. Therefore, a city-wide traffic model that consists of three relational algorithms, is proposed to identify significant links of the road network by using macroscopic fundamental diagram (MFD) as traffic dynamic characteristics. Firstly, weighted-traffic flow and density extraction algorithm is provided with simulation modeling and regression analysis methods, based on MFD theory. Secondly, critical links identification algorithm is designed on the first algorithm, under specified principles. Finally, threshold algorithm is developed by cluster analysis. In addition, the algorithms are analyzed and applied in the simulation experiment of the road network of the central district in Hefei city, China. The results show that the model has good maneuverability and improves the shortcomings of the threshold judged by human. It provides an approach to identify critical links for actual traffic management and intelligent control, and also gives a new method for evaluating the planning and design effect of the urban road network.

Keywords urban road network, critical links, intelligent transportation system, macroscopic fundamental diagram

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1 Introduction

Links are components of urban road network systems that significantly affect network performance. Specifically, the occurrence of traffic congestion and incidents in relevant sections reduces the operating efficiency of regional road transport networks and increases travel delays and costs. Therefore, scholars have extensively studied models of critical city-wide links.

Generally, *critical links* [1–3] are defined as the sections with the most significant effect on the connectivity and transmission of road networks. The methods and models of critical link identification are different in various research and application areas. Most methods focus on network topology and vulnerability to explore the static structural characteristics of road networks. Critical links are identified with important section indicators to evaluate network accessibility and reliability. Taylor et al. [4] proposed a heuristic method to find significant road sections using a probabilistic approach. According to this study, an increase in the number of sections as a result of a probabilistic approach leads to superior relative utility and seriously influences network performance. The author also utilized the loss of community amenity to analyze the network vulnerability of major roads. Jenelius et al. [5] established an approach to identify significant sections on the basis of increasing the indicators of average travel time on a network for each pair of origin-destination (OD) when the weighted demand of links is closed. The approach was applied to the road network calculation of northern Sweden. Scott et al. [6] analyzed the deficiency of the volume/capacity indicator in critical link identification and proposed the network robustness index indicator to evaluate overall system performance. Qiang and Nagurney [7] developed a method for measuring unified network performance to assess the importance of links and nodes. Ji [8] defined traffic capacity, road network topological structure, security and recurrent congestion frequency, and time balance factors of traffic flow as attributes for the identification of bottleneck links based on a rough set. Sullivan et al. [9] identified and sorted significant sections according to different link capacity-block values. This method quantifies the robustness of the transport network. Thereafter, several scholars [10,11,2,3] proposed the relative accessibility index, critical section index for the assessment of network topology vulnerability, total disruption delay time, and the variation index for the average traveling distance to identify critical links. Such models are commonly used to identify critical links from static road networks with traffic parameters of individual sections. However, they are insufficient for reflecting the relevance of individual sections and the overall road network with dynamic traffic characteristics. Therefore, these approaches may be suitable for transportation planning, but they cannot be applied to traffic dynamic management and intelligent control problems.

Aside from identifying critical links on the basis of network topology and static traffic characteristics, other studies have explored the identification of critical links with image technology and MFD theory. Schintler et al. [12] identified key links and nodes with the image technology of a geographic information system based on the grid and analyzed road network resiliency via complex network theory. Such an approach is still used to identify key sections on the basis of static traffic data. Xu et al. [13] established a road network simulation in Zhuhai District, Guangzhou City. Key sections of the road network were selected by observing and comparing the variations in MFD. Such an approach involves the use of MFD theory to identify key sections but not the mathematical model. Moreover, traffic managers still use empirical values to determine the threshold of identifying key sections and traffic state. For instance, Ma et al. [14,15] analyzed a large-scale transportation network with deep learning theory and used an artificial threshold (20 km/h) to divide traffic state into congested and non-congested states. However, the threshold calculation does not involve the use of a scientific method. Therefore, a scientific and reasonable method for determining threshold is urgently needed.

Although existing works have attempted to identify critical links, they provide insufficient support for traffic managers to rapidly discover the most critical links in a dynamic transportation network and thereby implement strategies to mitigate traffic congestion. Therefore, the practical applications of critical link identification technology are limited.

The objective of this work is to develop an approach to identify critical links from dynamic transportation networks for practical application in traffic management and control. To reflect dynamic traffic characteristics, we introduce MFD theory because of the following advantages:

1) MFD data are dynamic; thus, the traffic state of a road network is dynamic.

2) The state of a road network and a single road is relevant.

Therefore, on the basis of MFD theory and the previous works on identifying critical links, we analyze extensive experimental and actual MFD data to establish an MFD regression model and critical link identification model. Moreover, we construct a road network traffic simulation model based on real road network traffic data to inspect the critical link identification model. Consequently, we provide a method for extracting critical links from a dynamic transportation network and an approach for evaluating the effect of road network planning.

According to the research objective and scope described above, this paper is organized as follows:

1) Section 2 briefly introduces MFD and its calculation methods.

2) Section 3 describes critical link identification model in detail. The maximum weighted traffic volume of MFD is chosen as an index to calculate the threshold and to construct critical link identification algorithms on the basis of MFD theory and regression and cluster analysis methods.

3) Section 4 illustrates experimental results and the implementation process. Algorithms are analyzed and verified with traffic simulation experiments combined with actual network traffic data on the central district of Hefei City.

4) Section 5 presents conclusions and directions for future research.

2 MFD theory and calculation model

Daganzo and Geroliminis [16–18] proposed the early concept of MFD. MFD is described as the relationship between the traffic volume and the density of a road network and the relationship between area weighted volume and the total traffic amount of a network. MFD represents the network traffic state as the relationship between the number of moving vehicles in the road network and the network operation level. Daganzo and Geroliminis [16–18] analyzed and demonstrated the existence of MFD using the traffic simulation data of the San Francisco business district (USA), that of Nairobi (Kenya), and the measured traffic data of Yokohama (Japan).

On the basis of the description of MFD, Zhang et al. [19] further developed the theory of MFD. Relying on a stochastic cellular automaton model, they compared the MFD of arterial road networks, which were governed by three adaptive traffic signal systems, under several boundary conditions. Therefore, the authors identified the “hysteresis phenomenon” of MFD under the traffic signal conditions.

Aside from the verification of the existence of MFD and the “hysteresis phenomenon,” the calculation model of MFD has also been also provided. Let i (i belongs to A) and l_i represent a road segment between adjacent intersections and its length, respectively. Let A represent the set of road segments. According to MFD theory, an MFD is formed and can be calculated with the following equation (Eq. (1)).

$$\begin{cases} Q = \sum_i k_i l_i, \\ q^w = \sum_i q_i l_i / \sum_i l_i, \\ q^u = \sum_i q_i / \sum_i l_i, \\ k^w = \sum_i k_i l_i / \sum_i l_i, \\ k^u = \sum_i k_i / \sum_i l_i, \\ k_i = o_i / s, \end{cases} \tag{1}$$

where Q, q^w, and k^w are the total traffic volume, weighted volume, and weighted density of the road network, respectively, and q^u and k^u are the unweighted volume and unweighted density of the road network, respectively. q_i, k_i, and o_i are the volume, density, and time occupation rate of the link i, respectively. s is the average effective length of the vehicle that is generally set to 5.5m.

However, relatively few studies have explored the extraction of the critical links of a dynamic transportation network with MFD theory. In the present study, we use MFD theory to reflect the dynamic traffic characteristics of the relationship between links and the road network. To obtain a well-defined MFD, the “hysteresis phenomenon” effect is neglected.

3 Critical link identification model

3.1 Definition of critical link

To meet the requirements of dynamic traffic management and control, we consider critical links as the set of all links that can significantly affect the traffic state of a road network when the links are removed from or added to a particular road network. Therefore, critical links are defined as

$$N(x) = \{x | \eta_x(\Delta k_x, \Delta q_x) \geqslant r, x \in D\}, \tag{2}$$

where x is the critical link. η_x is the distance value between the extreme points of the original road network MFD and the new road network MFD when a link is removed. r is the threshold value. D is the set of links of the entire road network.

3.2 Method

Our main objective is to extract critical links with significant effects on the state of the dynamic transportation network based on a well-defined MFD. We therefore develop a mechanism to meet the following goals:

1) To neglect the “hysteresis phenomenon” effect and minimize the variance in link densities to guarantee a well-defined MFD of a transportation network.

2) To extract a set of links with significant effects on the smooth-congested state of the road network.

3) To develop a threshold determination method with statistical analyses instead of human judgment.

On the basis of the aforementioned goals, we design a critical link identification mechanism that consists of three consequent algorithms. First, we build an MFD shape model and extract the weighted traffic flow and weighted traffic density through a regression analysis based on MFD data. In this step, we produce a network traffic simulation environment with our own simulation software while minimizing the difference in the traffic densities of adjacent networks according to the measured network traffic data on the central district of Hefei City, China. Moreover, a multiple nonlinear regression algorithm is utilized to determine a favorable MFD curve that can extract the major parameters reflecting the smooth-congested state of the road network. Second, we compare the variable values of the MFD weighted volume and unweighted density before and after the removal of the links from the road network to extract the critical links. In this step, we use the Pythagorean theorem and comparative analysis to analyze the moving distance of the inflection point of the smooth-congested state before and after the link removal. Finally, we arrange the moving distances of the inflection points of the smooth-congested state for each link in descending order to identify the threshold of the critical links. In this step, k-cluster analysis, which can efficiently classify sample data, is utilized to determine the threshold. The threshold of the critical links is the value between the first cluster and the second cluster. Moreover, a number of clusters, which show good maneuverability, can be inputted according to the needs of traffic management and control or those of road network planning. In addition, our approach is examined through a simulation of an actual road network in Hefei, China.

The critical link identification mechanism and relevant models are described in the following sections.

3.2.1 Weighted traffic flow and unweighted density extraction algorithm

A large volume of traffic simulation and real data show that the basic shape of the MFD curve approximates a quadratic curve distribution. As shown in Fig. 1(a), Daganzo [16] illustrated the MFD of a business district in San Francisco (USA) with simulation data. As shown in Fig. 1(b), Geroliminis and Daganzo [18] displayed the MFD of a business district in Yokohama (Japan) with real floating car data. As shown in Fig. 1(c), Xu et al. [13] displayed the MFD of the Zhu Hai District in Guang Zhou City (China) with simulation data. As shown in Fig. 1(d), Geroliminis et al. [20] displayed a three-dimensional vehicle MFD (3D-vMFD) points for bi-modal traffic with simulation data. These MFD figures indicate the existence of a regular pattern of the traffic state of a road network from free flow to congested flow. Moreover, the inflection point of the traffic state conversion can be calculated via statistical analyses.

On the basis of previous studies on MFD, we assume that the basic shape of the road network MFD is quadratic. Therefore, the MFD distribution curve is fitted with the multivariate nonlinear regression analysis method. Through this method, the regression relationships of the functions between the dependent variables and the multiple independent variables are established through statistics based on observation data. However, building and calculating the multivariate nonlinear regression model of the road network MFD is difficult. Therefore, the multivariate linear regression model is utilized with the following equation (Eq. (3)) to reduce the difficulty of model construction:

$$\begin{cases} y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon, \\ y = q^w, \\ x_1 = (k^u)^2, \\ x_2 = k^u, \end{cases} \tag{3}$$

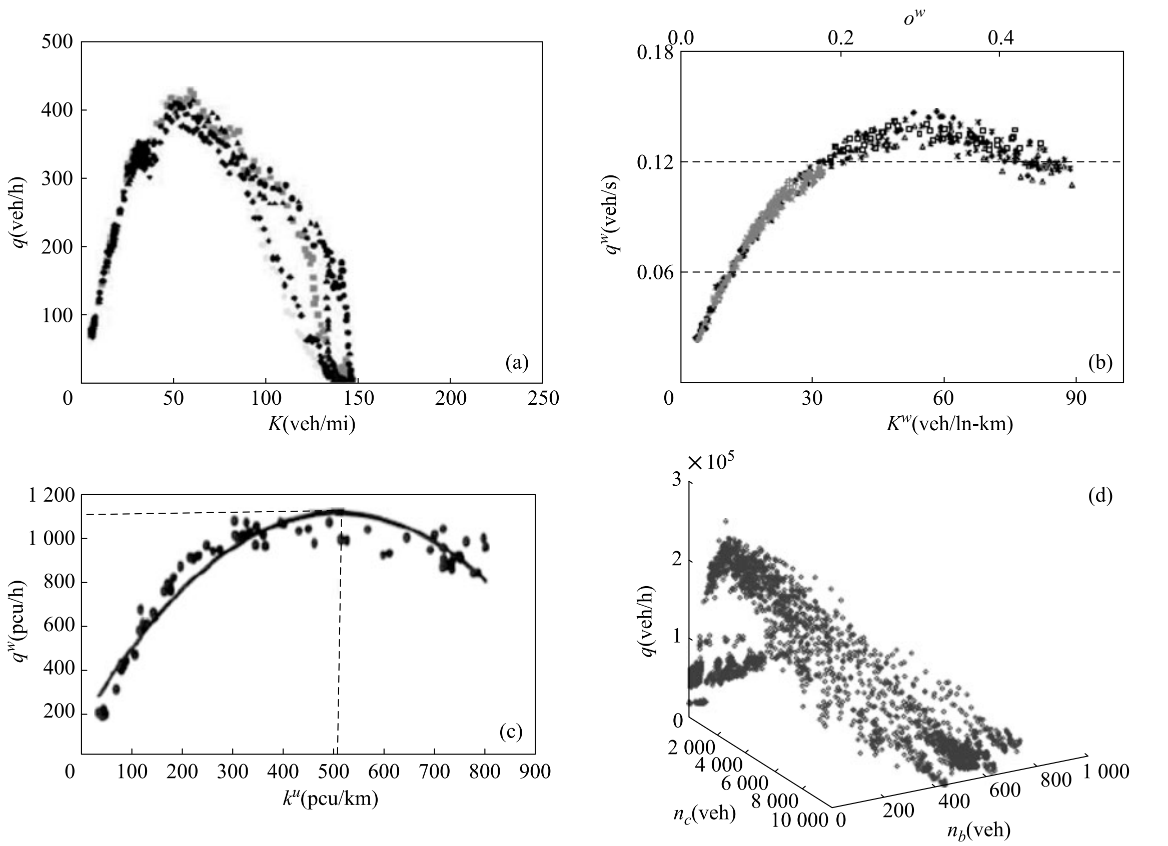

Fig. 1 MFD curves from previous findings. (a) MFD in San Francisco (USA); (b) MFD in Yokohama (Japan); (c) MFD in Zhu Hai district of Guang Zhou City (China); (d) The 3D-vMFD points for bi-modal traffic

where y is equal to the traffic weighted volume of the road network. β_0, β_1, and β_2 are the regression coefficients, which can be calculated with the least squares method according to the experimental data. x_1 and x_2 are equal to the unweighted traffic density squared and the unweighted traffic density of the road network, respectively. ε denotes the random error.

Calculation becomes relatively simple when we turn the MFD distribution curve into the multivariate linear regression model. However, the regression coefficients β_0, β_1, and β_2 need to be approximated to make the model resemble the experimental road network MFD curve closely. Therefore, we set R^2 ($R^2 \leqslant 1$) as the optimized fitting coefficient to test the regression effect. A large R^2 equates to a good fit. Moreover, the regression model is close to the MFD distribution curve of the road network. R^2 can be calculated with the following equation (Eq. (4)):

$$R^2 = \frac{\sum_{i=1}^{N} (\widehat{y_i} - \overline{y})^2}{\sum_{i=1}^{N} (y_i - \widehat{y_i})^2} + \sum_{i=1}^{N} (\widehat{y_i} - \overline{y})^2. \tag{4}$$

After the best fitting model is achieved with a large proportion of the experimental traffic data, Eq. (3) can be turned into the multivariate nonlinear regression model of the road network with the following equation (Eq. (5)):

$$q^w = \beta_0 + \beta_1 (k^u)^2 + \beta_2 k^u + \varepsilon. \tag{5}$$

The traffic states of the road network evolve from the smooth state to the congested state at point (k_0^u, q_0^w) of the MFD curve, where k_0^u and q_0^w are the extreme unweighted density and weighted volume of the road network, respectively. Therefore, we consider the point (k_0^u, q_0^w) as the inflection point of the network traffic state transition and select it as a significant factor to judge the effect of key sections on the traffic state of a road network. From Eq. (5), we can deduce the extreme unweighted density and weighted volume of the network MFD with the following equation (Eq. (6)).

$$\begin{cases} k_0^u = -\dfrac{\beta_2}{2\beta_1}, \\ q_0^w = \dfrac{4\beta_0\beta_1 - \beta_2^2}{4\beta_1}. \end{cases} \tag{6}$$

3.2.2 Critical link identification algorithm

The first step in identifying the critical links from dynamic traffic characteristics is the construction of the weighted traffic flow and unweighted density extraction algorithm. After completing the first step, we identify an appropriate method to construct the critical link identification algorithm model. Several rules are developed to identify critical links.

1) Critical links are identified by comparing the changes in traffic states before and after the addition or removal of a link in the road network. The changes in traffic states are determined with the variations in the extreme unweighted density and weighted volume of the road network. The variation in the extreme unweighted density of the road network can represent advancement or delay when the road network reaches a congested state, which is denoted as Δk_i. The variation in the extreme weighted volume of the road network can represent the change in the total traffic volume of the road network, which is denoted as Δq_i.

2) We assume that the total traffic OD demand of the road network is consistent when adding or removing a link in the road network.

3) We set the functions Δk_i and Δq_i as variations of the extreme unweighted density and weighted volume of the road network, respectively. If the value of the function exceeds a predetermined threshold value, the link i is judged as a key section of the road network.

To obtain the maximum degree of the links that affect the whole road network traffic state, we must consider Δk_i and Δq_i. Therefore, we choose a straight line distance between the inflection points before and after removing the section as an important indicator to generate the function, which can be calculated according to the following equation (Eq. (7)):

$$\begin{cases} \eta_i(\Delta k_i, \Delta q_i) = \sqrt{\Delta k_i^2 + \Delta q_i^2}, \\ \Delta k_i = k_0^u - k_{i0}^{u'}, \\ \Delta q_i = q_0^w - q_{i0}^{w'}, \end{cases} \tag{7}$$

where $\eta_i(\Delta k_i, \Delta q_i)$ is the variation of the extreme unweighted density and weighted volume of the road network. k_0^u and q_0^w are the extreme unweighted density and weighted volume of the original road network, respectively, and $k_0^{u'}$ and $q_0^{w'}$ are the extreme unweighted density and weighted volume of the road network, respectively, when adding or removing the link i from the original road network.

Critical links can be identified through the following equation (Eq. (8)):

$$\begin{cases} \eta_i(\Delta k_i, \Delta q_i) \geqslant r, \text{ the link } i \in \mathrm{C}; \\ \eta_i(\Delta k_i, \Delta q_i) < r, \text{ the link } i \notin \mathrm{C}, \end{cases} \tag{8}$$

where r is the threshold value and C is the set of critical links of the urban road network. Δq_i and Δk_i can be calculated through the experimental traffic simulation data.

From this step, all sections are classified into two sets according to Eq. (8). One set is the set of critical links, and the other set is the set of non-critical links. The purpose of identifying key sections from the dynamic traffic characteristics is achieved in this step. However, the critical link identification algorithm is not perfect. Traffic managers frequently judge the threshold value r instead of performing a rational scientific calculation. To compensate for this deficiency, we focus on establishing the threshold algorithm on the basis of cluster analysis.

3.2.3 Threshold algorithm

In a complex network evolution mechanism, road grade emergence is the intrinsic property of the road network and the inevitable result of the self-organization of network traffic [21,22]. On the basis of the idea of road grade emergence, we classify the links of the road network with cluster analysis to avoid the inaccurate determination of the threshold value by traffic managers. The reasons for applying the cluster analysis method in the threshold determination are as follows:

1) Cluster analysis is a scientific classification method based on statistical data.

2) Cluster analysis avoids the arbitrary and inaccurate threshold determined by traffic managers.

3) Cluster analysis is quick and efficient.

Based on cluster analysis, the main calculation rules of the threshold algorithm are as follows:

1) We arrange $\eta_i(\Delta k_i, \Delta q_i)$ in descending order to form an ordered set, which is recorded as $\{\eta_i(\Delta k_i, \Delta q_i)\}$.

2) The threshold value r is extracted by classifying $\{\eta_i(\Delta k_i, \Delta q_i)\}$ with cluster analysis. A sample data set is divided into k clusters from the beginning of the initial division. The criterion function is optimized with a repeat control strategy. Each cluster can be represented by the centroid point or the object closest to the center. We use the k-means clustering algorithm to improve the computational efficiency of road classification because the algorithm achieves good scalability and fast convergence. In the algorithm, each cluster is represented by a centroid. The remaining objects are allocated to the most similar cluster in terms of the distance between the objects and the centroid of the cluster. Then, the new centroid of each cluster is repeatedly calculated until the criterion function is converged.

According to the grade level of urban road network planning or intelligent traffic control requirements, we assume that traffic managers divide links into k clusters to extract key links. k is set to be the same as the number of grade levels or controlling levels of urban road planning to facilitate the optimal adjustment of road grade or the control of critical sections preferentially. Although the required manual intervention to determine k is minimal, the threshold calculation method is still better than the previous threshold r determined by human experience.

The critical links extracted by our model are the set of links with the greatest influence on the traffic state of the road network. After k is determined, the threshold value r can be calculated as follows:

1) Select the centroid $c_1, c_2, \ldots, c_k$ as the initial cluster centers.

2) Assign each object to the cluster with the smallest distance between the object and the cluster. Each cluster is represented by the mean of all objects. For each point $v_i(v_i = \eta_i(\Delta k_i, \Delta q_i)$, find a centroid c_j to achieve the minimum distance between v_i and c_j. Then, assign v_i to the j group.

3) Recalculate the centroid c_j of each group after all points are assigned to the appropriate group.

4) Repeat steps 2 and 3 until the divided data do not change.

5) Find the boundary value of the first class and second class on the basis of the results of the cluster analysis. The threshold value r is equal to the boundary value. The following is an explanation for why we choose the boundary value of the first class and second class as the threshold value: we define the links with the greatest impact on the traffic state of the road network as the key sections; the links that belong to the first class show the strongest effect on the network traffic state.

This calculation method of the threshold value improves scientific calculation and retains the rationality of human intervention because the initial classification depends on traffic managers.

3.3 Boundary conditions

When the critical link identification model is implemented, several rules should be followed:

1) The critical link identification model is constructed with MFD theory and the calculation model. Therefore, the network traffic system must satisfy the conditions of MFD existence proposed by Daganzo, that is, the entire road network should be either congested or clear.

2) The “hysteresis phenomenon” effect is not considered when we fit the MFD curve of the road network.

3) The implementation principle is that the impact of traffic state is estimated when removing or adding a section of

the road network. The total effect is neglected when deleting or adding multiple sections.

3.4 Other instructions

Although the identification model of critical urban links with MFD comprises three algorithms and several parameters, their effect on the final result is not satisfactory because of the following reasons. MFD is a unique transport property of the road network in the macroscopic traffic model, which involves numerous parameters. In the study of traffic models, algorithms and numerous parameters combined by a basic transportation model and other algorithms are frequently used. Other studies showed that the use of multiple algorithms and parameters has little influence on the final results. For instance, Ji et al. [23] proposed three algorithms and numerous parameters to separate urban transportation networks in space on the basis of the MFD model. Ma et al. [15] proposed a long short-term memory neural network (LSTM NN) model to predict traffic speed; the traffic model and LSTM NN model both involve a large number of parameters.

4 Implementation

4.1 Implementation procedure

We conduct the following procedure to apply and compare the effects of the critical link identification model.

1) Obtain the traffic flow, speed, and density data of each link on the basis of the GPS data of taxis and the analysis data of video detectors in the network. We retrieved the input data of our simulation network from the GPS data of 2 000 taxis and the analysis data of 300 video detectors. The GPS and analysis data covered the entire Hefei central network from April 6, 2016 at 0:00 to April 7, 2016 at 0:00.

2) Build a traffic simulation model and experimental environment with traffic simulation software on the basis of the transportation infrastructure data of the central district road network in Hefei City. The traffic simulation software includes the dynamic traffic assignment (DTA) model, which scientifically ensures the reallocation of traffic flow in the same OD condition. Therefore, when link i is deleted in the same OD conditions, the experimental data can be exported from the traffic simulation model and utilized to analyze critical links with the identification model.

3) Fit the MFD curve, and extract the set of inflection points (k_0^u, q_0^w) for each link of the simulation road network with the weighted traffic flow and density extraction algorithm on the basis of the experimental data.

4) Calculate Δk_i, Δq_i, and $\eta_i(\Delta k_i, \Delta q_i)$ with the critical link identification algorithm.

5) Arrange $\eta_i(\Delta k_i, \Delta q_i)$ in descending order to obtain the threshold value r by employing the threshold algorithm with computer calculation.

6) Compare the results of the critical link identification with the road network planning schematics to verify the identification effect and analyze the differences.

4.2 Simulation road network description

The road network used for the simulation is located in the central district of Hefei City (see Fig. 2), which belongs to Ahhui province of China. The road network has an area of 800km^2, and it includes 203 intersections and 354 sections with lengths ranging from 111m to 1600m. The number of two-way lanes per section ranges from two to eight. The free speed of the traffic flow is about 30km/h. The traffic signal system comprises a given cycle control of multi-phases, and the cycle time often ranges from 60s to 180s.

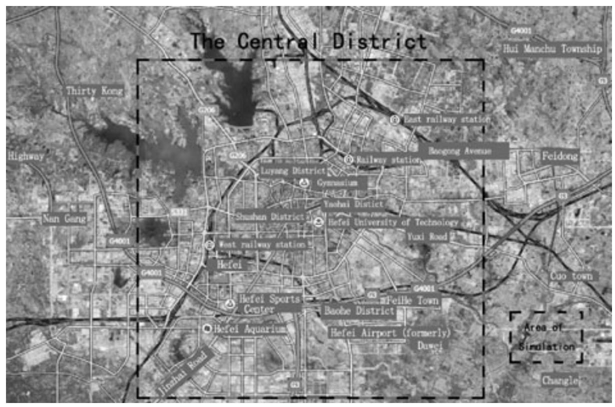

Fig. 2 The central district location in Hefei city and the area of simulation road network

The data of the simulated experimental road network approximate the actual road network data. The simulation time is 24h with 5min intervals.

4.3 Results from the experimental data

4.3.1 MFD of the simulation road network

First, the traffic flow, average speed, and density data of each link are obtained from the GPS data of 2 000 taxis and the analysis data of 300 video detectors in the central network of Hefei City. The 24h data were retrieved from two different sources (April 6, 2016):

• Fixed sensors: 300 video detectors located in the middle of the arterial lanes in the area with 5min traffic flow counts and average speed measurements.

- Mobile sensors: 2 000 taxis equipped with GPS or other positioning devices, which reported vehicle position and other data, including time stamps.

The weighted volume q^w per hour and its unweighted density k^u are calculated with Eq. (1) on the basis of the real data of the central district of Hefei City. Figure 3 shows the scatter plots of q^w-k^u and demonstrates the existence and basic shape of an MFD for the simulation road network.

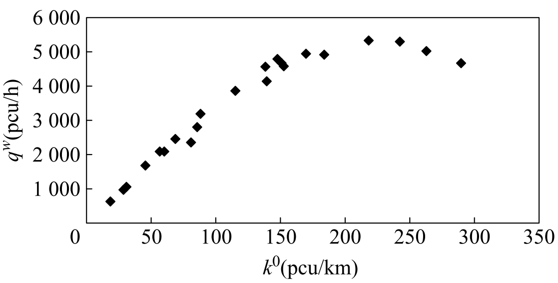

Fig. 3 The analysis result of the entire central district network MFD of Hefei city (q^w-k^u curve): weighted average volume vs. un-weighted average density

Second, the simulation road network is designed according to the actual central district network of Hefei City to obtain the reallocated weighted average volume and unweighted average density of the new road network using the DTA model when link i is deleted. Detectors are set up in each link of the simulation road network to aggregate the mean speed of all vehicles, the traffic flow, and other relevant traffic data with 5min intervals. Figure 4 shows the scatter plots of $q^{w'}$-$k^{u'}$ of the new network MFD when link *1* is deleted.

In the same manner, we obtain all $q_i^{w'}$-$k_i^{u'}$ of the new network when the link i alone is removed from the simulation network.

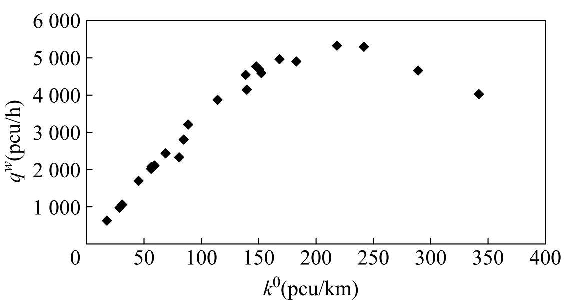

Fig. 4 The analysis result of the new central district network MFD of Hefei city when link *1* was deleted ($q_i^{w'}$-$k_i^{u'}$ curve): weighted average volume vs. un-weighted average density

4.3.2 Results of weighted traffic flow and unweighted density

After verification of the existence of MFD in the central district network of Hefei City, the MFD curve is fitted to extract the weighted traffic volume and unweighted density with the weighted traffic flow and density extraction algorithm.

In this step, the scatter points of q^w and k^u are analyzed via regression analysis with computer calculation.

Figure 5 depicts the MFD fitting curve, fitting coefficients, and optimized coefficient.

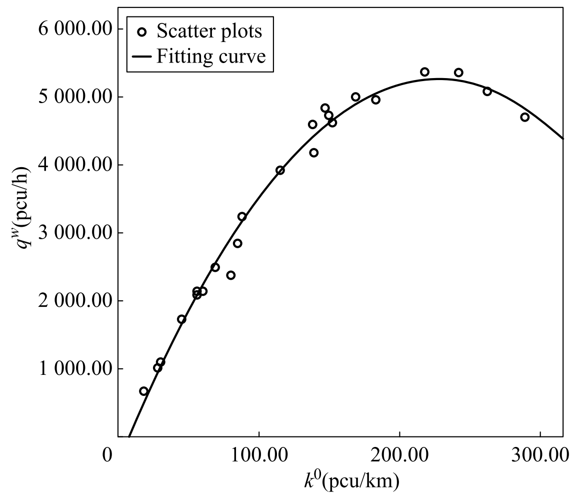

Fig. 5 MFD fitting curve, fitting coefficients and optimized coefficient of simulation network ($\beta_0 = -351.632, \beta_1 = -0.11, \beta_2 = 49.654, R^2 = 0.988$)

From Fig. 5, we can obtain the best fitting coefficients β_0, β_1, and β_2. According to Eqs. (3) and (5), the best fitting curve of the MFD for the simulation road network can be represented by the following equation with $R^2 = 0.988$:

$$q^w = -351.632 - 0.11(k^u)^2 + 49.654k^u + \varepsilon. \tag{9}$$

Using Eq. (6), we calculate the extreme weighted traffic volume and unweighted density as the following values (Eq. (10)) according to the previous equation of the MFD curve for the simulation network:

$$\begin{cases} k_0^u = 226 \text{ pcu/km}, \\ q_0^w = 5265 \text{ pcu/h}. \end{cases} \tag{10}$$

Similarly, according to the data in Fig. 4, we obtain the weighted average volume and unweighted average density of the new MFD as the following values when link *1* is removed:

$$\begin{cases} k_{10}^u = 229.96 \text{ pcu/km}, \\ q_{10}^w = 5292.64 \text{ pcu/h}. \end{cases} \tag{11}$$

With the same method, we obtain the other weighted average volume and unweighted average density of the new MFD when link i is deleted.

4.3.3 Results of the threshold and critical links

According to the definition of critical links in this work, Δq_i, Δk_i, and $\eta_i(\Delta k_i, \Delta q_i)$ are calculated with Eq. (7) to obtain the critical links, which can significantly influence the traffic state of the road network.

Figure 6 illustrates the distance between the original network and the new network when link *1* is removed.

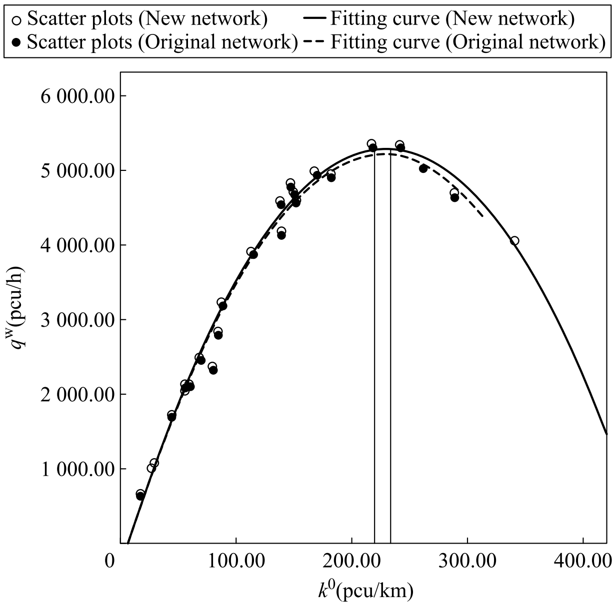

Fig. 6 The distance $\eta_1(\Delta k_1, \Delta q_1)$ between the original network and the new network when link *1* was removed

Figure 6 shows that the distance between the original network and the new network when link *1* is removed is 28. When all the distances $\eta_i(\Delta k_i, \Delta q_i)$ are calculated, $\eta_i(\Delta k_i, \Delta q_i)$ are sorted in descending order to form the sequence set $\{\eta_i(\Delta k_i, \Delta q_i)\}$.

In China, the urban road network is generally divided by the Planning Bureau into four grades. Therefore, we set the cluster number to 4. Comparing the classification results with the road grade of road network planning is convenient.

Finally, Fig. 7 depicts the classification results, which are the outputs based on the computer calculation.

Figure 7 shows that 60 critical links with significant effects on both the total traffic capacity and traffic state variation are identified for the simulated road network. Furthermore, we can obtain a threshold value of 127.8. Moreover, we find links with little impact on the whole road network.

4.3.4 Comparative analysis of the results of our model and the road network plan

From Fig. 7, we obtain the critical links of the simulation road network identified with the proposed critical link identification model. To compare the proposed model with the urban road network plan, we use the Hefei Urban Master Plan (2006–2020) from the Hefei Municipal Planning Bureau website. Figure 8 depicts the urban road grade of the central network of the plan.

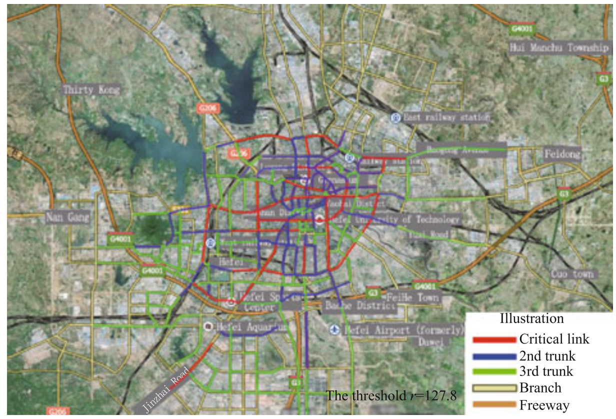

Fig. 7 Critical links defined by $\{\eta_i(\Delta k_i, \Delta q_i)\}$ of simulation experimental network

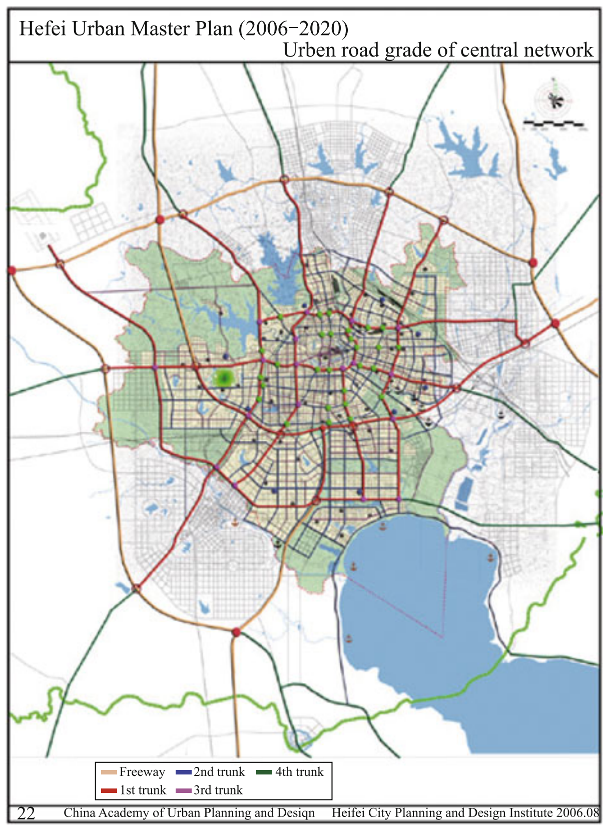

Fig. 8 Urban road grade of central network from the Hefei Urban Master Plan (2006–2020)

In the plan, 140 trunks significantly influence the whole simulation network. However, the critical links of the plan and the results of the proposed model are not fully consistent.

By comparing Figs. 7 and 8, we find that 47 critical links are consistent with the Hefei Urban Master Plan (2006–2020), whereas most of the remaining sections are fewer than those in the plan.

This finding may be explained as follows. Fig. 7 depends on the actual traffic OD and dynamic traffic data. The results are based on the real detected data, which differ from the expected traffic OD. By contrast, Fig. 8 depends on the expected traffic OD. The expected OD is obtained through predictions of the traffic planning model, which contains artificial data analyzed via artificial experience.

Therefore, a significant gap exists between our results and the planning results. Obviously, the results of our model are close to actual traffic conditions. Traffic managers can quickly and conveniently identify intelligent traffic control sections to be prioritized from the dynamic traffic state. The significant gap in the road classification indicates the difference between the actual OD and forecast OD. Our critical link identification model compensates for the shortcoming of the road classification divided by the expected OD of the artificial forecast. Furthermore, the practical data and scientific optimization method of our approach can support urban planners in adjusting road grade classification. Thus, our results provide a new method for evaluating adjustments in the road level plan of an urban road network.

5 Conclusions and discussion

In this work, we introduced MFD theory and proposed a critical link identification model with consideration of dynamic traffic characteristics. We investigated the actual data of the central district network in Hefei City, including the road fundamental data, traffic parameter data, and signal timing of the network. We constructed the simulation network according to the investigation data and extracted experimental data. We analyzed the real detected data and experimental data with mathematical statistics software and finally obtained the results related to the critical links. We found that the critical links of the road network plan and those of our model were not fully consistent. Our model showed that most of the critical links exert the most significant effect on the total traffic capacity and traffic state variation. Our model thus allows traffic managers to quickly identify priority control segments. Furthermore, our model provides a new method for evaluating the planning and design of urban road networks from the perspective of dynamic traffic characteristics. On the basis of our results and our comparison with the road network of the Hefei Urban Master Plan, we found that our model is adaptable to practical applications because of the variable cluster and threshold algorithm for traffic management variability.

However, our model requires the following conditions:

1) The entire network should be either congested or clear.

2) The “hysteresis phenomenon” effect is not considered when fitting the MFD curve of the road network.

3) The impact of traffic state is estimated when removing or adding a section of the road network.

In our next study, our model can be improved by analyzing the influence of multiple sections on the entire network and by simplifying the calculation through dimension reduction.

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